

Behavior of Eccentric Rotors Through the Critical Speed Range

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Abstract. This paper presents a new view of a key overlooked phenomenon when dealing with significantly eccentric rotors, namely the switch of the rotor's axis of precession and consequent orientation in its bearings while passing through the critical speed region. This occurs in conjunction with torque effects unique to the case where a rotor's principal mass axis and torque input axis are not coincident. This condition also governs the rotor's phase shift process. Around the critical speed, the inertia from the eccentric mass becomes sufficiently large as to alter the mode of rotation, bringing the rotor toward a "state of least action", where the precessional orbit rapidly decreases, and the rotor begins to rotate about its principal mass axis. The most immediate benefit of recognizing this behavior is in the development of a new balancing method pertaining especially to flexible, bowed or eccentric rotors, designed for use in balancing facilities.

Keywords: phase shift, critical speed, eccentricity, torque, inertia

1 Introduction

From practical experience through balancing hundreds of rotors with significant body eccentricities, and observing discrepancies in rotor behavior between the balancing facility (uncoupled) and in an installed rotor train, it is evident that highly eccentric rotors exhibit unique behavior and require unique treatment. Among the observations are an apparent change in bearing position and rotor orientation when passing through the critical speed region, and notable shaft-centerline hysteresis between acceleration (with torque) and coasting/deceleration (without torque). From these observations and supposition of rotor behavior, the author has developed a new balancing method to reliably resolve these cases, often requiring many fewer runs than standard approaches. This method has been proven in practice numerous times with the most "difficult" highly flexible, highly eccentric rotors. A detailed description of the method, named the Quasi-High Speed Balancing Method (or 2N+1 method), can be read in associated papers by the author. [1]

Knowing that the method works, the next challenge is to explain why, and this paper is an attempt to better describe the unique physical behavior of eccentric or bowed rotors when passing through the critical speed region. The term "region" is added since the behavior being addressed begins at first induced deflection of the rotor and the initiation of phase lag, and continues until the speed range where the phase lag has reached 180 degrees. Above this speed as well are some additional unique behaviors only seen on eccentric rotors. The descriptions here are geared toward working engineers more so than academic interests, hence the explanations are entirely conceptual and visual, and are not intended to be rigorous.

The standard, simplified approach of rotordynamics study that follows from an oscillation-based model of vibration using spring, mass and damping parameters, in particular the Jeffcott model, cannot completely describe all true physical events that occur on real-life rotors while passing through a rotor/bearing system critical speed. A key overlooked phenomenon when dealing with eccentric rotors is the switch of the rotor's axis of precession and natural re-alignment in its bearings while passing through the critical speed region. This occurs in conjunction with a subtle dissociation of the angular velocity of rotor spin and rotor precession, as the relative angles of rotor deflection response and reactive centrifugal force diverge (in what is typically considered phase lag), even independent of system damping. Around the critical speed peak, the inertia from the eccentric mass, driven by input torque, becomes sufficiently large as to alter the mode of rotation, bringing the rotor toward a "state of least action", where the precessional orbit rapidly decreases, and the rotor begins to self-align and alter its rotation from its geometric axis (the line connecting the journal centers) to a rotation/precession about its principal mass axis.

In the classical view, after passing the critical speed region, dynamic motion/precession for all cases is assumed to remain centered about a single consistent non-rotating reference axis common through the speed range. In our view, the initial geometric axis defines the rotation center and shaft orientation up to the critical speed peak, while the dynamic motion/precession above the critical peak is centered (and constrained) about the principal mass axis, which defines the new shaft centerline position, which is determined by the rotor's state of least action and the position it assumes in space in the bearings.

Ultimately, it is the change in axis and bearing position and mode of precession from the effects of inertial forces that especially creates problems for standard balancing methods when applied to eccentric, flexible rotors. The goal of balancing an eccentric rotor then is really to prevent the change in precession axis from noticeably occurring, which prevents the change in rotor orientation in the bearings. This is achieved by bringing the mass axis coincident to the geometric axis from the start, and allowing the rotor to accelerate to speed without deflection or distortion. In balancing eccentric rotors, it is crucial to create this condition to avoid unknowingly installing and constraining a rotor to run in a state other than the one in which it was balanced and aligned in the bearings.

2 Features of an Eccentric Rotor

The unique feature of a rotor with significant distributed eccentricity is that it encompasses quasi-constrained, forced non-centroidal rotation of the full rotor, with a torque input axis not coincident to the principal mass axis. It can be considered "quasi-constrained" in that the constraints/forces that force non-centroidal rotation are eventually overcome, but not until around the critical speed peak. (These aren't purely physical contact constraints, but are combined with the effect of internal forces and moments.) This is in contrast to the behavior created by typical local unbalance (like a chipped blade) on an otherwise concentric rotor, which produces a dynamic bending response at speed (or in resonance) within otherwise centroidal rotation at all speeds. Of course, such a dynamic or resonant response occurs as well in an eccentric rotor, in combination with or superimposed on the other effects.

The phase shift for a significantly eccentric rotor encompasses the process through which the rotor passes as it switches from constrained non-centroidal rotation/precession into centroidal rotation/precession while progressing through the critical speed region. Note that there is no “perfectly concentric” rotor in real life, and in reality all rotors will have at least some small eccentricity, or separation between their geometric axis and mean mass axis. ISO 1940 acts as an effective distinguishing measure of eccentricity below which the separation in axes (and resulting switch in precession center) can be neglected.¹ In rotors within ISO 1940 tolerances, the phase shift process still occurs, but the shift from a non-centroidal to centroidal condition happens without measurably affecting the rotor behavior, since the geometric and mass axes are nearly coincident from the start. Likewise, if the centrifugal force from such a small eccentricity remains unable to generate any noticeable deflection, the rotor will be observed as simply passing through the critical speed region without deflection response or precession orbit.²

The combined rotor-bearing system can be thought of as comprising two interconnected equilibriums, one being the non-rotating “static” equilibrium between the shaft and the bearing, and the other being the rotating “quasi-dynamic” equilibrium of the forces in the spinning rotor itself. Each of these equilibriums affects the other, and instability in one can produce instability in the other. The non-rotating, “static” equilibrium remains generally stable in position (only following the shaft centerline path), held by gravity load which is constant, and oil hydrodynamic forces which govern the rotor’s elevation in the bearings. The rotating “dynamic” equilibrium remains referenced to the geometric center of the shaft or its neutral centerline. The dynamic forces from rotor rotation/precession are vectorially summed with these static forces, with the net summation then governing the position and orientation of the rotor in its bearings. In an eccentric rotor, the internal moments and forces dynamically generated by the eccentric mass will at some sufficient speed overcome the gravity load forces and alter and govern the rotor’s orientation in its static bearing equilibrium.³

The generated “vibration” of a rotor is not a single simple motion, and is not truly modeled by a standard linear mass-spring-damper system, since the “vibration” of the rotor itself is really a translational motion in a precession orbit, without any real oscillation of the shaft (if the orbit is circular). That said, there still is a standard oscillatory component within the total rotor-bearing system.⁴ Though the “vibrating” rotor is really precessing or translating, its bearing support (oil film primarily, but also the shell,

¹ A major OEM developed similar standards in which runout/eccentricity limits are approximately twice those in ISO 1940. By the author’s experience, this is an appropriate and suitable adjustment, as the ISO 1940 limits are more conservative than is necessary in practice

² In this case, an accurate determination of an orbit “high point” may be nearly impossible, and the phase angle between this and a shaft mark is not necessarily a reliable measurement

³ This implies that the root of bearing instabilities, including subsynchronous whirl and oil whip, really arises from the condition of the rotor itself and not bearing design, though clearly with proper bearing type and design, the system can “withstand” and stabilize a much wider breadth of dynamic behavior from rotor imperfections.

⁴ In the standard classical view, rotor “vibration” is considered acting in the line of the bearing force response, which then can be presented as a linear spring-mass-damper system, but it is erroneous to assume this visualization as the behavior of the shaft itself.

housing and pedestal) can be viewed as a linear, non-rotating spring, with an effective line of action matching the attitude angle of the rotor in the bearing.

Historically, phase lag is considered to originate entirely in system damping, primarily via damping in the oil film, though it is still questionable if damping alone represents the full true physics of the phenomenon [2]. This correlates to viewing the rotor motion as a linear spring-mass-damper system, or the summation of two such linear systems 90 degrees apart, much in a manner in which displacement sensors interpret rotor motion. However, through a detailed look at rotor forces and behavior, an additional mechanism of phase lag is also seen, with its foundation in the conservation of angular momentum within rotor precessional motion. This mechanism of phase lag is only relevant and only active during the transient acceleration of a rotor, and does not have an effect at steady state.

3 Behavior and Forces of an Eccentric Rotor

To explain the phase shift process and the switching of the axis of rotation, assume for instance a flexible, significantly eccentric rotor (constrained) in oil-film bearings that operates above its first critical, with a distributed eccentricity with a peak near the midplane tapering to none at the endplanes. To more easily recognize the interaction of the “static” and “dynamic” equilibriums, assume the “static”-related forces as located in a radial plane at the bearing(s), and the “dynamic”-related forces as located on a radial plane at the rotor midplane.

The progression of the rotor’s behavior through its operating speed range can be conceptually divided here into four sections. The first is from initial roll to the point where phase shift initiates, the second and third cover the critical speed region from 0 to 90 degrees and from 90 to 180 degrees of phase shift respectively, and the fourth covers the speed region from just beyond the completed phase shift progression.

3.1 The Effect of Torque

A distinction can also be made regarding applied torque to an eccentric rotor, as compared to an equivalent concentric rotor. The total torque required to accelerate an ideally concentric rotor to a given speed considering its total inertia (ignoring other losses in this example) can be called “basic torque”. Since an equivalent but eccentric rotor will produce a larger moment of inertia, both from its innate eccentric mass or runout, and from its larger resulting precession orbit, the extra torque required to accelerate this rotor to the same given speed can be called “supplemental torque”. Conceptually speaking, the total torque applied can be divided into these two components, with the “supplemental torque” creating a “torque moment” around the mass axis, contributing to a number of effects unique to eccentric rotors. Ultimately, all rotor behavior described is created, directly or indirectly, through torque input.

Torque is applied to a rotor at its geometric axis at all speeds, either symmetrically about the blades of a turbine, or symmetrically about the coupling, like in a generator rotor, which is coupled symmetrically about its geometric axis. Therefore, in an eccentric rotor, there is a distance between the geometric axis through which the torque is applied, and the radial center of mass of any given “slice” of the rotor. Since the principal mass axis (or inertia) of the rotor body is at a position different than the axis of the input torque, there will be a “torque moment”, or reaction at the point of torque input

from the inertia of the rotor body, present at each radial plane of the rotor where eccentricity exists. This “torque moment” is present through all rotor speeds and grows proportionally as torque itself increases, both through increased speed and increased unit load. (This effect leads to the observed behavior named “torque whirl” by J. Vance). [3]

A familiar example of this situation is drilling a hole vertically into one end of a long flat board sitting on a floor when suddenly the drill bit jams, though the drill motor is still applying torque. This applied torque will immediately turn the entire board, with a pivot point at the location of the drill bit. However, because the true center of mass of the board is somewhere near the center of its length, when the far end of the board begins to rotate, the total inertia of the board will react against this applied force, as the board naturally will tend to turn about its mass center point, and the near end of the board will immediately jerk as it “pushes” tangentially against the drill. However, if a strong constraint is holding the drill in place, then the board will maintain its turning around the torque input point, unless this constraint force is overcome. If there is little or no constraint force, torque is still applied at its input point and still turns the board, while at the same time, the inertia of the board also turns the torque input point around the board’s center of mass.

In a similar manner, the reaction to this moment from the applied rotor torque acting away from the center of mass would tend to naturally turn or flip the rotor over in the direction of rotation and attempts to move the torque input axis in an orbital path around the principal mass axis. In other words, the rotor will want to naturally rotate about its center of mass, not its torque input axis, if these points are not coincident.

3.2 Behavior, Forces and Moments through the Rotor Speed Progression

From the point of initial roll, a flexible rotor will be in a state of gravity sag (even if pre-conditioned for a time on turning gear), where the rotor actually spins following the sag line, with a tension-compression cycle in the rotor material. In this region, from zero-speed up to a few hundred rpm (or higher, depending on the flexibility and length-to-diameter ratio of the rotor), the eccentric mass will produce a subtle oscillation or wobble in the rotor body, while maintaining a sagged state. The only significant forces are the input torque to accelerate the rotor, gravity acting on the rotor and the corresponding reaction forces in the bearings. The “torque moment” acts at this stage, but simply “tumbles” around with the rotor, since without any induced rotor bending, it has no notable effect.

At some eventual speed, the internal damping of the rotor material will prevent the necessary rapid switching of tension and compression along the sag line, and the rotor will “lock up” and begin to precess as a rigid, eccentric or bowed beam around a straight-line axis connecting the journal centers (the geometric axis). This continues until the speed when entering the critical speed region where phase lag initiates. Because of the strong constraint of gravity holding the journals down in the bearings, the rotor cannot switch its rotation to its mass axis in this speed range, and so is forced to maintain non-centroidal rotation. Because of this, and because the center of mass point itself is actually precessing now in some orbit around the geometric axis, a reactive centrifugal force will arise from this eccentric mass. This centrifugal force is exerted by the eccentric mass on the bearings/constraints, with the shaft acting as a means of connection between this force and the constraints. This acts to “pull” and elastically bow the entire rotor body outward, maximized at the midplane. Initially, the reactive

centrifugal force and the resulting bending deflection are in line with each other. This pulling acts against the internal spring of the rotor (which resists the bowing near the midplane of the length of rotor body), as the bearings constrain the ends from deflecting. The initiation of this section of rotor behavior corresponds with the beginning of the “critical speed region”, which also corresponds with the initiation of phase lag, with the response (or orbital) high point now lagging relative to this reactive centrifugal force that causes it.

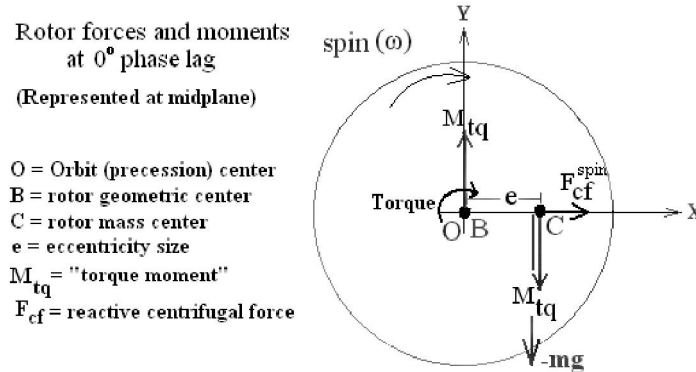


Fig. 1. Forces on an eccentric rotor at low speeds, prior to any induced deflection or phase lag.

Once in this state, and from here forward, it is important to note that a “vibrating” rotor no longer actually oscillates only in the manner of typical linear mass-spring vibration, but rather precesses in an orbit with the same side facing out (for synchronous motion), with no further tension-compression cycle of the rotor material (at least for circular orbits, though elliptical orbits will have some tension-compression). This motion can be broken up into a spinning around the geometric axis through which torque is always centered, and a synchronous translational precession orbit. Since the eccentric mass at this point remains still asymmetric to the precession of the rotor, it continues to generate an increasing reactive centrifugal force that “pulls” the eccentric rotor into an increasingly larger orbit. At the same time, the “torque moment” described earlier is still present, and still attempts to flip the rotor to rotate about its center of mass, but is now also resisted by a moment generated by a reactive centrifugal force from the precessional motion (in the line O to C) acting about the geometric center of the rotor.

To clarify the talk of moments, note that the utilization and description of moments here is not in the same manner as that used in engineering analysis, where multiple moments must be analyzed about the same common point to calculate a net single moment. Rather, the moments are described in a manner easier to recognize and visualize, and represent separate moments acting about different points, and more so represent the rotor’s natural tendency of motion than any external applied forces. In aggregate, these moments about different points do not create a net torque, but rather result in a net deflection, even though each individually would otherwise create a rotation. Combined, both this “torque moment” that would “flip” the rotor about its principal mass axis, and the “centrifugal force moment” act in tandem to maintain the forced non-centroidal

rotation of the eccentric rotor, and in the process contribute to the increasing induced shaft deflection.

Following the conservation of angular momentum, an increasing orbit will proportionally slow its angular velocity. Since torque is still applied symmetrically about the geometric center, the spin of the rotor continues at a rate that subtly disassociates from the orbital precession, which is slowed by comparison. The amount of total deflection of the rotor determines the relative rate of decrease in angular velocity of the precessional motion versus the spin. Conceptually speaking, the “basic torque” is applied to generate the spin of the rotor, while the “supplemental torque” must be applied to account for the added radius of precession, including the “torque moment” resulting from the eccentric mass.

Note that the true concept of conservation of angular momentum is applied in a closed system, and since there is a constant applied torque to the geometric axis, even at steady state, this isn't truly a closed system. Therefore, conservation of angular momentum doesn't fully apply here in the universal sense, but more during a particular set of conditions. The change in phase lag angle occurs only during active transient rotor acceleration and precession orbit growth. Since the spin and precession are equivalently driven using a common drive torque, causing the spin directly and indirectly causing the orbit (via reactive centrifugal force), the torque input can be "normalized" and the two motions considered solely in a comparative manner as if it were a closed system. The same object with the same single input torque is in two rotational motions, with the only difference being their transient instantaneously diverging radius of rotation, during which the precession orbit comparatively lags in angular velocity relative to the spin.

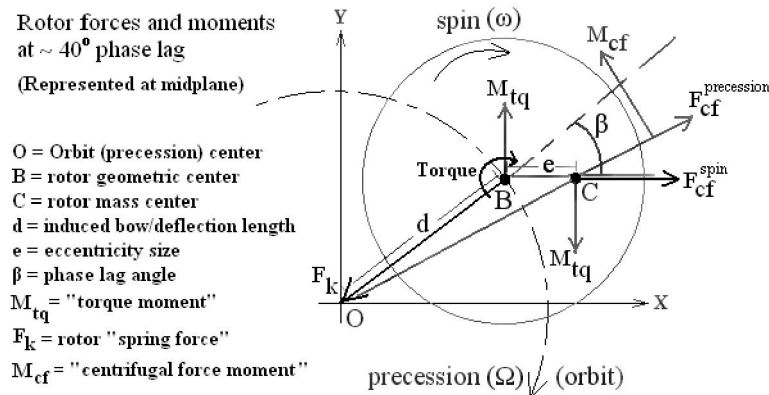


Fig. 2. Forces on an eccentric rotor in the critical speed region, before the peak response.

With a phase lag still less than 90 degrees, the rotor is effectively in two simultaneous, synchronous motions, the spin of the rotor around its geometric axis, B, and the precession of the rotor about its imaginary shaft centerline or orbital center O. The centrifugal force created by eccentric mass that is acting on and bending the rotor initiates from and continues from the spin of the rotor, pointing from the geometric center, B, toward the center of mass, C, while the resulting internal deflection/bowing, d, resisted by the “spring” of the rotor F_k follows the precessional orbit (which reacts

against the bearing constraints), with a high spot pointing outward from the precession center, O , through the geometric center, B . Additionally, the “torque moment” M_{tq} continues to tend to flip the rotor about its principal mass axis at C in the direction of rotation, while being countered by the “centrifugal force moment” M_{cf} acting through the center of mass, C , from the precessional motion, against the spin at B .

When the phase lag angle reaches 90 degrees, the reactive centrifugal force (from B to C) and the resulting deflection (from O to B) become orthogonal, and the centrifugal force no longer has any effective component remaining to maintain the rotor deflection. The remainder of the phase shift from 90 to 180 degrees, on the downslope of the critical speed response peak, occurs under a different mechanism than the way up. The potential energy that was stored in the deflected spring of the rotor is rapidly released into kinetic energy as the orbital radius from induced bending rapidly drops, and the rotor accelerates. Through the drop in rotor deflection and loss of effect from reactive centrifugal force, the remaining force constraint that had been preventing the rotor from rotating around its principal mass axis disappears, and the “torque moment” can finally flip the rotor to precess about its principal mass axis. This process shifts the phase angle between the direction of the heavy spot and the orbital high point the rest of the way to 180 degrees, and the rotor self-aligns in its bearings to the orientation of its principal mass-axis. At this phase lag angle with the “spin” 180 degrees ahead of the orbital peak, the mass center, C , becomes positioned between the geometric center, B , and the center of the precession orbit, O . In certain distributions or ratios of rotor eccentricity, the mass center and orbital center become nearly coincident. This instantaneous “least action” state corresponds to the sometimes-observed dip seen in measured displacement amplitudes either just before or after the critical speed peak.⁵

Though this previous description of phase lag only focuses on the rotor precession and torque effects on the rotor, there are also effects arising from oil film damping and internal hysteretic damping. The damping effects are more secondary however, and do not drive or govern the fundamental critical speed behavior, but rather affect certain response parameters such as the extent of the velocity range through which critical speed behavior is seen. Therefore, with all combined effects there is some variability in the observed timing of phase shift, along with corresponding amplitudes.

The final section of rotor behavior appears upon reaching the state of least action where the eccentric rotor is now precessing around its center of mass. Because there is still torque being applied to the geometric axis, the same “torque moment” from before continues to turn the geometric axis about the mass axis, still synchronously and without a constraint to its motion. At the same time, there is still a reactive centrifugal force from the spin of the rotor, pointing from the geometric axis, B , toward the mass center, C (effectively toward the center of precession, O), with the response to this force now 180 degrees behind, counteracting it. The combined result is a precession of the mass center, C , with an orbit radius equal in size to the amount of eccentricity, e . Simultaneously, the motion also incorporates the synchronous spinning of the geometric center,

⁵ This “amplitude dip” correlates to the “torque moment”-driven flip of the rotor, where the rotor briefly rotates about its principal mass axis in a “least action” state. For equivalent unbalance (oz-in), the dip’s occurrence is dependent on input torque combined with the *ratio* of eccentricity (distance) to eccentric mass (weight). This dip occurs at pre-critical speed when eccentric mass is large but its distance from the geometric axis is small, and at post-critical speed when the total eccentric mass is smaller but at a larger distance. [4]

B , around the mass center, C , which itself is seen as the “runout high point” in the amount of the eccentricity. This maintains the geometric center, B , in a constant outside position relative to the position of the mass center to the orbital center. The total peak amplitude therefore seen by a displacement sensor is equal to twice the eccentricity ($2*e$) while in this state of least action above the critical speed. This is in contrast to the measured peak amplitude at low speeds (essentially the rotor runout) being equal to the eccentricity ($1*e$). This displacement amplitude behavior can be verified by viewing real-life Bode plots for significantly eccentric rotors.

Once fully beyond the critical speed region, as the eccentric rotor is further accelerated, the ends are often driven into increasingly larger orbits, often out of phase and without bending deflection and without any notable phase shift. This is seen in Bode plots as an up-sloping amplitude line above the first critical, but with a steady phase angle, with the response often out of phase at each end.

The fact there is an increasing orbit, but without a notable phase shift, comes from the interplay between another “torque moment” and the tendency of the rotor now to remain in precession about its principal mass axis. If there is still residual eccentricity on the rotor it is in all likelihood biased toward one side, away from the axial center of gravity of the rotor. With the axial center of gravity now located on the current rotational axis, this point acts as a pivot and a constraint to the rotor motion. The axial center of the eccentric mass distribution generates an axial moment about the rotor’s overall axial center of gravity, driving a rocking motion of the rotor.

With continued added torque and rotor acceleration, a reactive centrifugal force will similarly be exerted radially by the residual eccentric mass on the bearings, and in the same manner as before, the shaft will act as a connection between this force and the bearings. Vibration displacements will increase proportionally to the square of speed. The rate of amplitude increase is greater if the rotor is flexible, or reaction forces in bearings will increase more if the rotor is more rigid. Both displacements and forces will be additionally amplified at the beginning of the next critical speed region.

Recall as well that the torque is still being applied around the geometric axis, even though the rotor is now precessing about its mass axis, and therefore, the torque moment described earlier is still present. The “centrifugal force moment” described earlier is also still present, but with a comparatively smaller effect due to the still small bending deflection radius at this state. Because the rotor is now already precessing about its mass axis, and because the centrifugal force is small, this “torque moment” is not resisted as in the overall state before the first critical peak, and so synchronously “tumbles” the rotor about its mass axis. In this state, the “tumbling” and the spin of the rotor (from input torque) remain synchronously driven and do not yet generate a new phase lag. This condition represents the natural, “least action” state of an eccentric rotor for the situation where torque is applied outside of the mass axis while precession occurs around the mass axis.

At some point, a speed is reached where the still-growing reactive centrifugal force is large enough to again elastically bend each half of the rotor individually. This now-larger centrifugal force, having induced some bending deflection, creates a new “moment” that again begins to resist the “torque moment”. These two forces (or “moments”) in combination cause a net outward “pull” as before, now on each half of the rotor separately. This creates the second critical speed region, with a new phase lag progression on each half of the rotor, and a second peak in amplitude. This second-critical progression progresses in each half of the rotor in a manner self-similar to the

whole rotor at lower speeds while it passed through the first critical speed region. In a Bode plot, amplitude rise and fall within the second critical region can sometimes be observed superimposed on top of the steady up-sloping amplitude line generated from the rigid rocking of the rotor described in the previous paragraphs, as both behaviors can occur simultaneously.

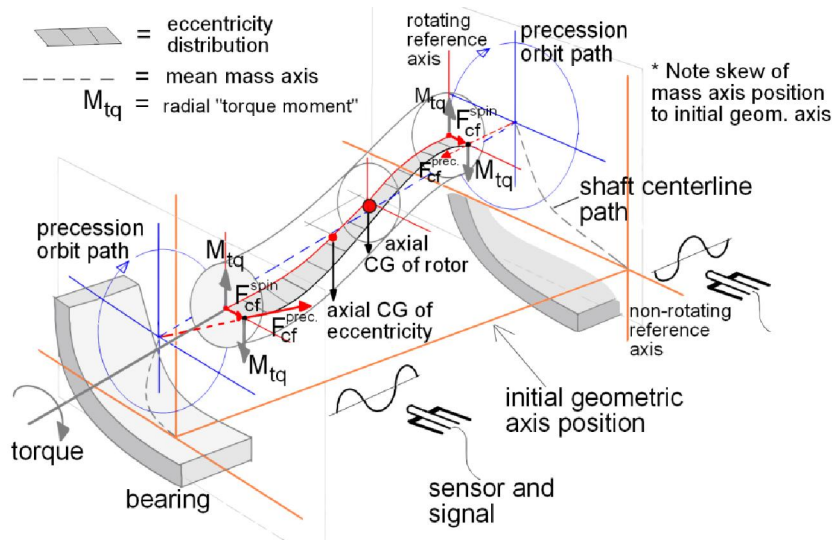


Fig. 3. Above the first critical, with precession about mass axis, and re-alignment in bearings

One additional observed behavior that is connected to the preceding descriptions is the hysteresis sometimes observed in the shaft centerline between the path up while accelerating an eccentric or bowed rotor and the path down while decelerating. The path down is followed without any torque input for the entire speed range back to standstill, and without torque (and the "torque moment") acting as a constraint forcing non-centroidal rotation on the way down, the rotor will precess about its mean mass axis through the entire deceleration. Since this is a different precession condition than during acceleration, the relative position of the rotor ends as seen at each bearing will be changed, with the hysteresis size being proportional to the amount of eccentricity. This creates the hysteresis loop sometimes seen between the accelerating and decelerating shaft centerline path.

4 Conclusion and Reconnection to Rotor Balancing

The long preceding description of the switch in rotor precession axis through the phase shift presents the unique behavior of flexible rotors with large distributed eccentricity, and provides a rationale for the requirement to balance such rotors in $2N+1$ balancing planes. From the tendency of eccentric rotors to self-align in their bearings to their principal mass axis above the first critical, the goal of balancing should be to bring the principal mass axis coincident to the rotor's geometric axis, without producing rotor

distortion or internal moments, thereby preventing these phenomena from occurring. By following the Quasi-High Speed Balancing Method using $2N+1$ balancing planes, any eccentric rotor can be brought to a smooth running condition when installed in a properly aligned rotor train.

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